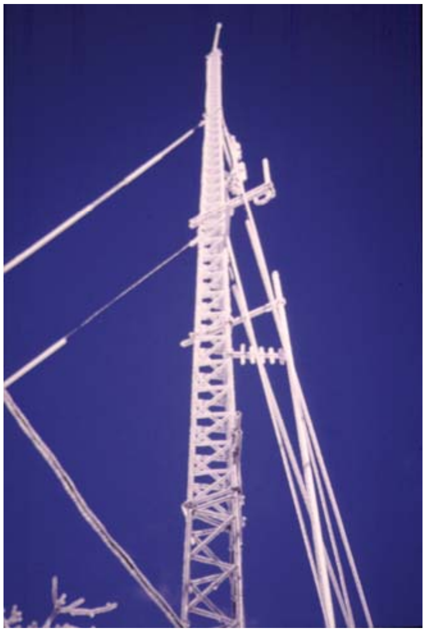
Fact Sheet

DETERMINING DISTANCE OF ICE FALL FROM TOWERS

You can determine the approximate distance from a tower at which a chunk of ice of any size might land. Because that distance is dependent on what you assume about the ice, and because, to our knowledge, no one has measured the frequency of ice chunks of different sizes and shapes falling from towers, it is appropriate to use simplifying assumptions to get an approximate analytical solution to the problem, rather than to develop a numerical solution with all the detailed aerodynamics.

Basically, as the ice starts falling from the tower, it falls faster and faster, accelerated by gravity until it reaches its terminal velocity, which depends on how much drag there is on it from the air through which it moves. You can determine the terminal velocity by equating the force of gravity with the drag force:



$$mg = \frac{1}{2} C_D \rho_a A V_T^2,$$

where

m is the mass of the chunk of ice;
g is the acceleration of gravity;
ρ_a is the density of air;
C_D is the drag coefficient of the chunk of ice;
A is the cross-sectional area of the chunk of ice;
V_T is the terminal velocity of this chunk of ice.

Solve for V_T, to get

$$V_T = \sqrt{\frac{2mg}{C_D \rho_a A}}.$$

You then want to know how far the ice is blown horizontally by the wind while it is falling. First calculate how much time (Δt) it takes to reach the ground, if the ice travels at terminal velocity all the way down.

$$V_T \Delta t = H,$$

where H is the height of the tower (assuming the ice falls from the top).

Assuming that the ice moves horizontally as fast as the wind blows, the ice will travel downwind a distance D before it hits the ground, where

$$D = W \Delta t,$$

where W is the wind speed. Putting it all together,

$$D = HW\sqrt{\frac{\rho_a C_D A}{2mg}}.$$

This says that the ice that falls from the top of the tower travels a greater distance before it hits if the tower is taller or if the wind speed is greater, as we all would have guessed without going through the equations. The distance also increases as the area assumed for the chunk of ice increases and the assumed mass decreases. If you think of a parachute compared to a rock, that makes sense, too.

This is a crude approximation, useful for crude assumptions about the ice. In reality, the ice spends more time than Δt falling from the tower, since it takes some time to accelerate to the terminal velocity, so D is an under-estimate of the real distance. On the other hand, the ice takes some time to accelerate horizontally to the wind speed,

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depending on its shape, roughness, and orientation as it falls, so D is an overestimate of the real distance. If we're lucky, the errors from those simplifications cancel each other. The drag coefficient C_D depends on the shape and surface roughness of the ice chunk and its orientation as it falls. Often, for bluff bodies, $C_D = 1$ is not an unreasonable assumption. A positive or negative lift force may also act on the ice chunk during its flight, which could either increase or decrease D.

This formula for D can be used in metric units with

H in m,
W and V_T in m/s,
m in kg,
$g = 9.8$ m/s^2,
$\rho_a = 1.3$ kg/m^3,
A in m^2,

to give D in m (m = meters, s = seconds, kg = kilograms). In English units, with

H in ft,
W and V_T in mph,
m in lb,
$g = 32.2$ ft/s^2,
$\rho_a = 0.081$ lb/ft^3,
A in ft^2,

D is in feet if you put in the conversion factors for the various units in the equation, which gives

$$D = \frac{5280}{3600} HW \sqrt{\frac{\rho_a C_D A}{2mg}} .$$

So, for example, take H = 500 ft and W = 50 mph and see how different assumptions about the ice chunk affect D:

$$D = 1300 \sqrt{\frac{C_D A}{m}} .$$

Assuming $C_D = 1$, and assuming the ice density is 57 lb/ft^3, gives

A *(ft^2)*	m *(lb)*	D *(ft)*
0.25 (3 in. by 12 in.)	1 (3/4 in. thick)	650
0.25	0.6 (1/2 in. thick)	839
0.5 (4 in. by 18 in.)	2.4 (1 in. thick)	593
0.27 (3.25 in. by 12 in.)	2 (1.5 in. thick)	478

It might also be reasonable to assume an ice density less than 57 lb/ft^3 to account for the possibility of rime ice, rather than glaze ice, forming on the tower. This would result in a smaller ice mass for a given-size ice chunk and thus a larger D.

So, we have four not-unreasonable assumptions for the shape and size of a chunk of ice that might fall from the tower, and the calculated distance away from the tower at which it hits the ground varies from 478 ft to 839 ft. This illustrates the difficulties in determining the ice fall radius for a tower.

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